

HOMEWORK SET 21: FD STATS AND CONDUCTING ELECTRONS

Due Monday, April 7, 2025

PROBLEMS FROM OR AFTER TZDII¹ AND TREX²

TRex 9.23) a) Silver has exactly one conduction electron per atom. Use the density of silver, ($1.05 \times 10^4 \text{ kg/m}^3$) and the atomic mass of 107.87 g/mole to find the density of conduction electrons in silver.

$$n_{e, Ag} = 5.86 \times 10^{28} \text{ e}^-/\text{m}^3$$

TRex 9.24) What fraction of electrons in a good conductor have energies between $0.90 E_F$ and E_F at $T = 0$? **HINT: USE THE FERMI-DIRAC ENERGY DISTRIBUTION FUNCTION (BELOW) AND INTEGRATE (YOU SHOULD BE ABLE TO DO THE INTEGRAL YOURSELF). YOU SHOULD GET 14.6%.**

$$n_{FD}(E)dE = \frac{3N}{2} E_F^{-3/2} E^{1/2} \frac{1}{e^{(E-E_F)/kT} + 1} dE$$

TRex 9.25 altered) a) Looking at silver in Problem 23, you found $n_{e, Ag} = 5.86 \times 10^{28} \text{ e}^-/\text{m}^3$. Calculate the Fermi energy and Fermi temperature.

b) Find the Fermi velocity and compare it to the Maxwell-Boltzmann rms velocity at 300 K.

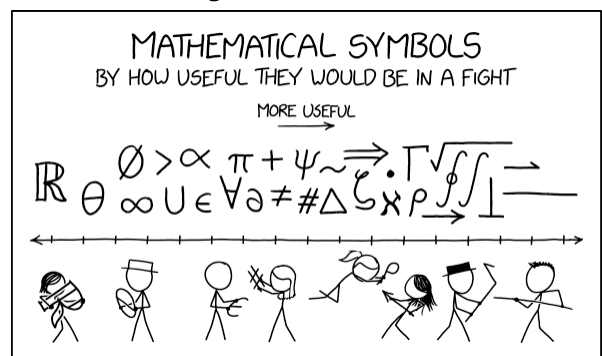
$$E_F = 5.50 \text{ eV}, T_F = 63,747 \text{ K}, v_F = 1.39 \times 10^5 \text{ m/s}$$

TZDII 13.48) a) Using Drude's formula $\sigma = ne^2\tau/m_e$ and the measured values of metal conductivities, calculate the collision time τ for silver (**GIVEN ABOVE**). Show that the units work.

MATERIAL	CONDUCTIVITY ($\Omega \cdot \text{m}$) ⁻¹	RESISTIVITY ($\Omega \cdot \text{m}$)
Silver	6.27×10^7	1.61×10^{-8}
Copper	5.88×10^7	1.70×10^{-8}
Aluminum	3.65×10^7	2.74×10^{-8}
Lead	4.8×10^7	21.0×10^{-8}
Stainless Steel	1.36×10^7	73×10^{-8}
<i>Metals (at 18°C)</i>		

b) Compute the thermal (MB) speed of an electron at 18°C. Drude assumed, incorrectly, that the mean speed of conduction electrons is given by their thermal speed. He also assumed, incorrectly, that it is the ions in a metal that scatter conduction electrons, which implies a mean free path of a few or several lattice constants (depending on the scattering cross section of the ions).

c) Use the thermal speed and the collision time to compute the mean free path. Your answer should be a distance equal to several lattice constants ($l_{Cu} = 0.3 \text{ nm}$), consistent with Drude's (incorrect) assumptions of atomic scattering. This calculation shows that, although Drude made two incorrect assumptions, his model was self-consistent



<https://xkcd.com/2343/>

¹ Taylor, Zafiratos, & Dubson, *Modern Physics for Scientists and Engineers*, 2nd Edition, Pearson, Prentice Hall, 2004

² Thornton and Rex, *Modern Physics for Scientists and Engineers*, 3rd Edition, Cengage Learning, 2013